Indian Statistical Institute
Mid-Semestral Examination
Topology IV - MMath II

Max Marks: 40 Time: 180 minutes.

Throughout, U, $V \ldots$ will denote open subsets in some Euclidean space.

(1) Construct the de Rham complex $\Omega^*(U)$ of U. Verify that this is a cochain complex. [10]

(2) Compute $H^0(V)$. [10]

(3) Let $A \subseteq \mathbb{R}^n$, $A \neq \mathbb{R}^n$, be a closed subset. Let $R : \mathbb{R}^{n+1} - A \longrightarrow \mathbb{R}^{n+1} - A$ denote the reflection $(x_1, \ldots, x_{n+1}) \mapsto (x_1, \ldots, x_n, -x_{n+1})$. Show that the induced homomorphism

$$R^* : H^{p+1}(R^{n+1} - A) \longrightarrow R^* : H^{p+1}(R^{n+1} - A)$$

is multiplication by -1 for all $p \geq 0$. [10]

(4) Given a closed subset $A \subseteq \mathbb{R}^n$ and distinct points $p, q \in \mathbb{R}^n$ we say that A separates p from q if p and q belong to different components of $\mathbb{R}^n - A$. Let A and B be two disjoint closed subsets of $\mathbb{R}^n$. Given two distinct points p, q in $\mathbb{R}^n - (A \cup B)$ show that if neither A nor B separates p from q, then $A \cup B$ does not separate p from q. [10]